Practice Midterm 1 Solutions

Thursday, February 24, 2022 7:26 PM

Math 2270, Midterm 1

October 8, 2021

PRINT YOUR NAME: Solns

Question	Points	Score
1	16	
2	12	
3	8	
4	9	
5	15	
6	20	
7	12	
8	8	
Total:	100	

- No advanced calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated.
- You have 50 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- Do the best you can!

1. (16 points) Find the general solution to the below linear system of equations if they are consistent. Write answer in parametric form. If not, write that there is no solution.

$$x_1 - x_2 - 3x_3 = 5$$
$$-2x_1 + 4x_2 + 8x_3 = -14$$
$$x_1 - 4x_2 - 6x_3 = 11$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 5 \\ -2 & 4 & 8 & -14 \\ 1 & -4 & -6 & 11 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -1 & -3 & 5 \\ 1 & -2 & -4 & 7 \\ 0 & -3 & -3 & 6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -3 & 5 \\ 0 & +1 & +1 & -2 \\ 0 & \cancel{1} & \cancel{1} & \cancel{-2} \end{array}\right]$$

$$x_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

2. (12 points) Is the vector $\vec{v}$ in the span of the other vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$?

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \vec{u_3} = \begin{bmatrix} -4 \\ 1 \\ 5 \end{bmatrix}, \qquad \vec{v} = \begin{bmatrix} 7 \\ -2 \\ -5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -4 & 7 \\ 0 & 1 & 1 & -2 \\ -1 & 3 & 5 & -5 \end{array}\right] \left[\begin{array}{ccc|c} 1 & -2 & -4 & 7 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -4 & 7 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

Row of all 0's then nonzero

— inconsistent!

No soln to $A\vec{x} = \vec{v}$, i.e. $\vec{v}$ is not in span u_1, u_2, u_3

3. (8 points) Which of the following sets of vectors are bases for $\mathbb{R}^2$? Circle either "Basis" or "Not basis" to the left of each set of vectors according to your answer.

Basis [✓ Not basis]

$$\left\{\begin{bmatrix}3\\-6\end{bmatrix},\begin{bmatrix}6\\-12\end{bmatrix},\begin{bmatrix}9\\-13\end{bmatrix}\right\}$$

3 vectors!

Basis [✓ Not basis]

$$\left\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\}$$

$\{\bar{0}\}$ is not lin indep.

Basis [✓ Not basis]

$$\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}$$

not enough vectors

[✓ Basis] Not basis

$$\left\{\begin{bmatrix}-2\\1\end{bmatrix},\begin{bmatrix}0\\-1\end{bmatrix}\right\}$$

$$\begin{bmatrix}-2&0\\1&-1\end{bmatrix}\xrightarrow{RREF}\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

4. (9 points) Determine whether each map is one-to-one, onto, both (isomorphism), or neither. Circle the appropriate answer, circling both if both are true and neither if neither are true.

[✓ one-to-one] [✓ onto] Rotation 120 degrees counterclockwise.

invertible.

[✓ one-to-one] [✓ onto] Multiplication by the matrix A, whose reduced row echelon form is B

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 12 & 1 & 20 \\ 11 & -3 & 18 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

RREF is Id.

one-to-one [✓ onto] Multiplication by the matrix A:

$$A = \begin{bmatrix} 1 & -2 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

REF has no zero rows

5. (15 points) Find determinants of the following matrices if possible or write that the determinant is not defined otherwise. Then figure out whether the matrix is invertible or not and circle the corresponding answer:

[✓ Invertible] Not Invertible

$$\det \begin{bmatrix} -2 & 5 \\ 8 & 13 \end{bmatrix} = -26 - 40$$

b/c det $\neq 0$

det = -66

Invertible [✓ Not Invertible]

$$\begin{bmatrix} -13 & 2 & 4 & 5 \\ 4 & 2 & -3 & 7 \\ 1 & 1 & -4 & 9 \end{bmatrix}$$

nonsquare matrices do not have det's!! & cannot be invertible!

det = ________________

Invertible Not Invertible

$$\begin{bmatrix} -2 & 1 & 0 \\ 3 & 20 & -4 \\ 10 & -5 & 1 \end{bmatrix}$$

$$-2 \cdot \underbrace{\det \begin{bmatrix} 20 & -4 \\ -5 & 1 \end{bmatrix}}_{0} - 1 \cdot \det \underbrace{\begin{bmatrix} 3 & -4 \\ 10 & 1 \end{bmatrix}}_{-37}$$

$$+ \cancel{0 \cdot \det \begin{bmatrix} 3 & 20 \\ 10 & -5 \end{bmatrix}}$$

det = 37

6. (20 points) Find a basis for the null space Nul A and column space Col A of the matrix A. Determine the dimensions of the null space and column space.

$$A = \begin{bmatrix} 1 & 1 & 3 & 5 \\ 2 & -2 & -6 & 2 \\ 0 & 2 & 6 & 7 \end{bmatrix}$$

Null Space Nul A:

$$\begin{bmatrix} 1 & 1 & 3 & 5 \\ 2 & -2 & -6 & 2 \\ 0 & 2 & 6 & 7 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & -4 & -12 & -8 \\ 0 & 2 & 6 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 & 5 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix} \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & \underset{\uparrow\ x_3 \text{ free}}{0} & 1 & 0 \end{array}\right]$$

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$$

Nul A basis: $\begin{bmatrix} 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$ dim Nul A: 1

Column Space Col A:

RREF has pivot col's 1, 2, 4

Col A basis: $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 7 \end{bmatrix}$ dim Col A: 3

7. (12 points) Find the inverse and determinant of the matrix

$$A = \begin{bmatrix} -3 & 5 \\ -7 & 9 \end{bmatrix}.$$

$\det A = -27 + 35 = 8$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \boxed{\frac{1}{8}\begin{bmatrix} 9 & -5 \\ 7 & -3 \end{bmatrix}}$$

= [illegible]

$$A^{-1} = \begin{bmatrix} \underline{\qquad} & \underline{\qquad} \\ \underline{\qquad} & \underline{\qquad} \end{bmatrix} \qquad \det A = \underline{\qquad\qquad}$$

8. (8 points)

1. Is the function $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$F\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} 1 - \boxed{ab} \\ a+b \end{bmatrix}$$

not linear!!

linear or not? Circle your answer.

F is: Linear [✓ Not Linear]

2. Let $F : \mathbb{R}^3 \to \mathbb{R}$ be the linear transformation

$$F\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = 3a + 2b + c.$$

If possible, find a matrix A such that $F(\vec{x}) = A\vec{x}$. If not, write that A does not exist.

$A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

$A =$ ____________________